7TH GKV TRAINING COURSE

MARCH 31, 2023 @ ZOOM

OVERVIEW OF GKV

GKV development team

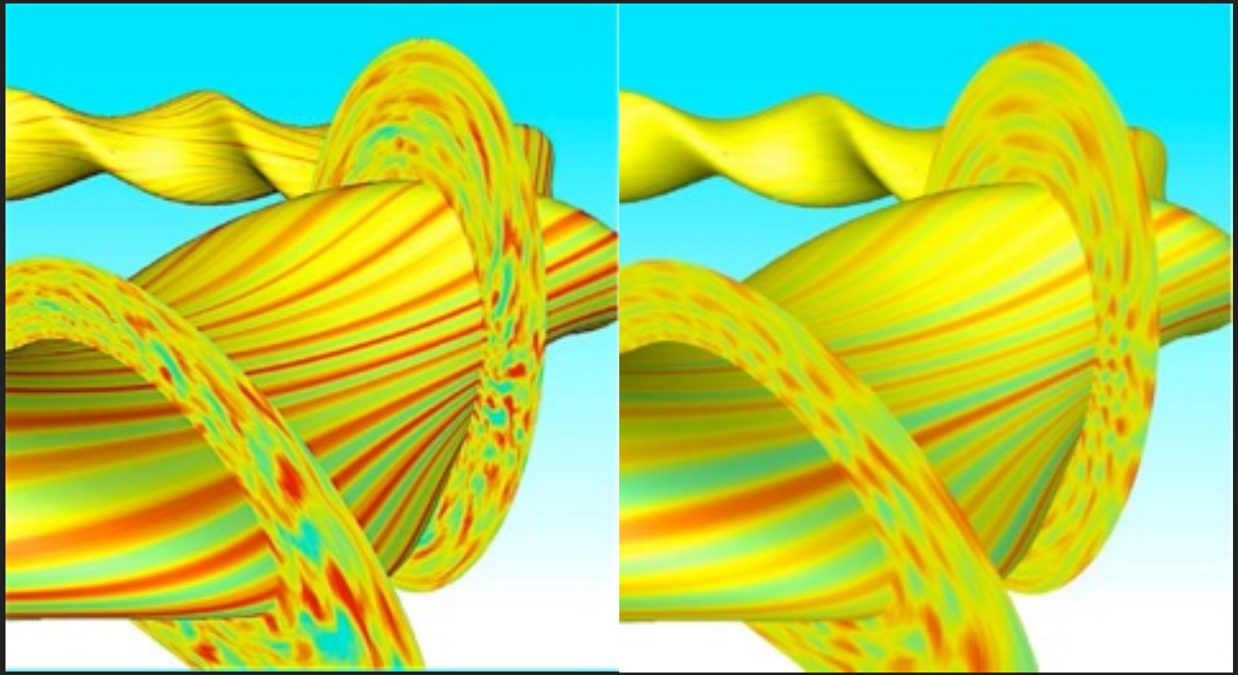
MARCH 31, 2023 @ ZOOM

GKV (GyroKinetic Vlasov) code

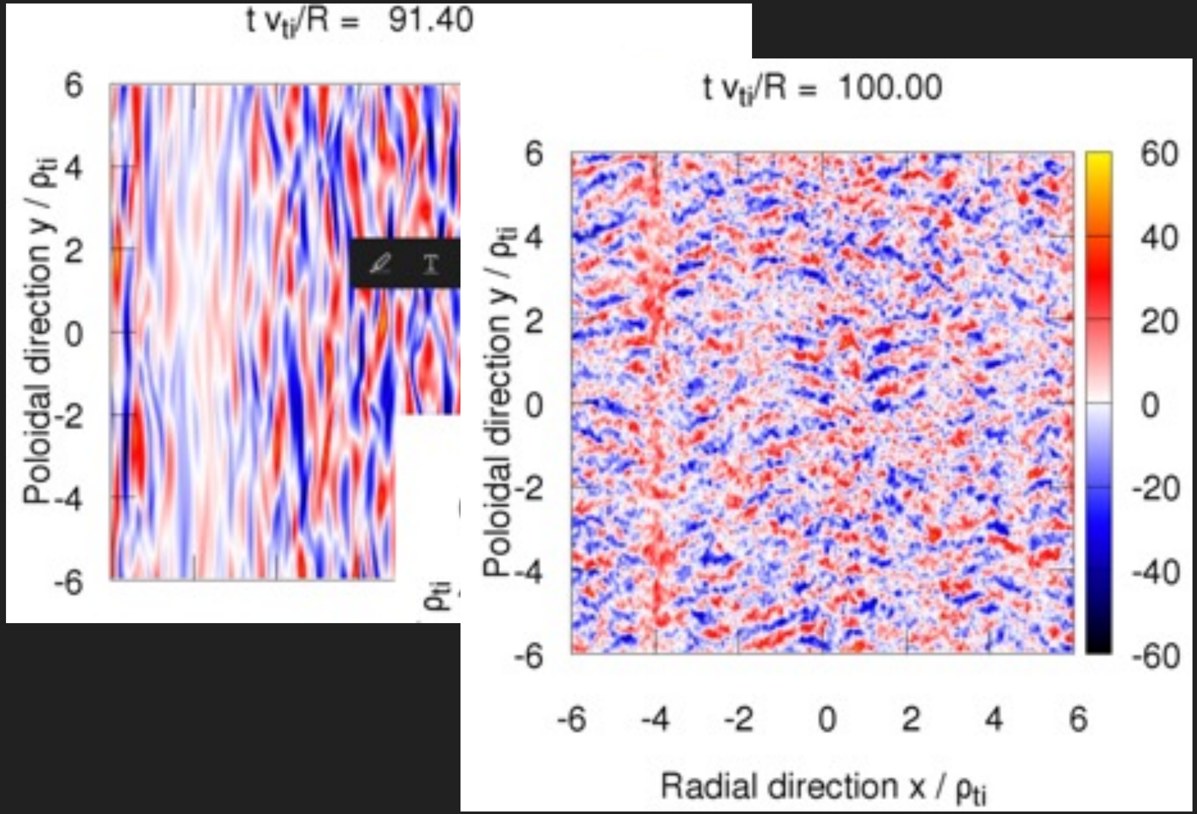
- Gyrokinetic simulation code
  - Tracking time evolution of turbulence and zonal flow in a magnetic field confined plasma
  - Local model with flux tube
  - Fix background magnetic field, density / temperature gradients
  - Deal with fluctuations
  - Electromagnetic field fluctuations and multiple particle species
  - Collisions between each particle species
  - Compatible with tokamak and helical geometries
  - Applicable for experimental magnetic field configurations
  - Interaction analysis with entropy balance

Application of GKV code

- Isotope effects in turbulent transport

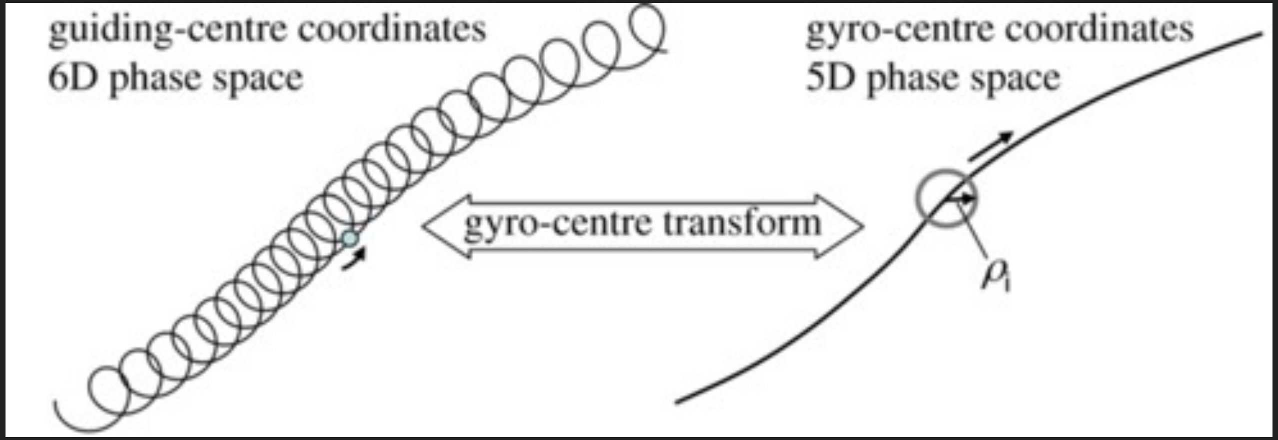


- Multi-scale turbulence



Gyrokinetic Theory

- Gyroscopic motion of charged particles is averaged to remove fast oscillating components
- Phase space coordinates are **reduced from 6 to 5 dimensions**



- Accurately treat (ballooning type) fluctuations with wavelengths of about the gyro radius in the direction perpendicular to the magnetic field and about the size of the device in the parallel direction
- Incorporate kinetic effects such as trapped and un-trapped particles, magnetic field drift, Landau damping, and finite gyro-radius effects

What phenomena does it apply to?

- Drift wave instability and drift wave turbulence
  - Ion/electron temperature gradient mode (ITG/ETG)
  - Trapped electron mode (TEM)
  - Kinetic ballooning modes (KBM)
  - Microscopic tearing mode (MTM)
- Zonal flow, geodesic acoustic modes (GAM)
- Kinetic Alfven wave
- Magnetic reconnection
- Damped/driven kinetic turbulence

What the current GKV code does not allow

- Relaxation of equilibrium distribution ⇒ Transport code
- Low (m, n) mode ⇒ MHD code
- Heating, particle supply ⇒ Fixed density and temperature distribution
- Non-linear effects due to parallel electric field
  ⇒ Eliminated by GK ordering

Equations solved in GKV 1
(Representation in wavenumber space)

$$\frac{\partial f_{sk}}{\partial t} + v_{||}\nabla_{||} f_{sk} + i\boldsymbol{k}\cdot\boldsymbol{v}_{sD} f_{sk} + N_{sk} - \frac{\mu\nabla_{||}B}{m_s}\frac{\partial f_{sk}}{\partial v_{||}}$$
$$= -\frac{e_s F_{sM}}{T_s}\left[v_{||}\left(\nabla_{||} J_{0sk}\phi_k + \frac{\partial J_{0sk} A_{||k}}{\partial t}\right) + i\boldsymbol{k}\cdot\boldsymbol{v}_{sD} J_{0sk}\phi_k - i\boldsymbol{k}\cdot\boldsymbol{v}_{s*} J_{0sk}(\phi_k - v_{||}A_{||k})\right]$$
$$+ \sum_{s,} C_{s,s'}(f_{sk}, f_{s'k})$$

$$\left[k_\perp^2 + \frac{1}{\varepsilon_0}\sum_s \frac{e_s^2 n_s}{T_s}(1 - \Gamma_{0sk})\right]\phi_k = \frac{1}{\varepsilon_0}\sum_s e_s \int J_{0sk} f_{sk} dv^3$$

$$k_\perp^2 A_{||k} = \mu_0 \sum_s e_s \int v_{||} J_{0sk} f_{sk} dv^3$$

Equations solved in GKV 2
(Abbreviations)

$$\nabla_{||} = \frac{1}{B\sqrt{g}} \frac{\partial}{\partial z}$$

$$\boldsymbol{k} \cdot \boldsymbol{v}_{sD} = \frac{m_s v_{||}^2 + \mu B}{e_s} \left(K_x k_x + K_y k_y \right)$$

$$\boldsymbol{k} \cdot \boldsymbol{v}_{s*} = -\frac{T_s}{e_s} \left[\frac{1}{L_{ns}} + \left(\frac{m_s v_{||}^2}{2T_s} + \frac{\mu \mathrm{B}}{T_s} - \frac{3}{2} \right) \frac{1}{L_{Ts}} \right] k_y$$

$$N_{sk} = -\frac{1}{B} \sum_{\boldsymbol{k}' + \boldsymbol{k}'' = \boldsymbol{k}} \left(k_x' k_y'' - k_y' k_x'' \right) J_{0sk'} \left(\phi_{k'} - v_{||} A_{||k'} \right) \left(f_{sk''} + \frac{e_s F_{sM}}{T_s} J_{0sk''} \phi_{k''} \right)$$

$$F_{sM} = n_s \left(\frac{m_s}{2\pi T_s} \right)^{3/2} \exp\left(-\frac{m_s v_{||}^2}{2T_s} - \frac{\mu B}{T_s} \right)$$

Equations solved in GKV 3
(Abbreviations cont.)

$$J_{0sk} = J_0(k_\perp \rho_s)$$

$$\Gamma_{0sk} = I_0(k_\perp^2 \rho_{ts}^2)\exp(-k_\perp^2 \rho_{ts}^2)$$

$$k_\perp^2 = g^{xx} k_x^2 + 2g^{xy} k_x k_y + g^{yy} k_y^2$$

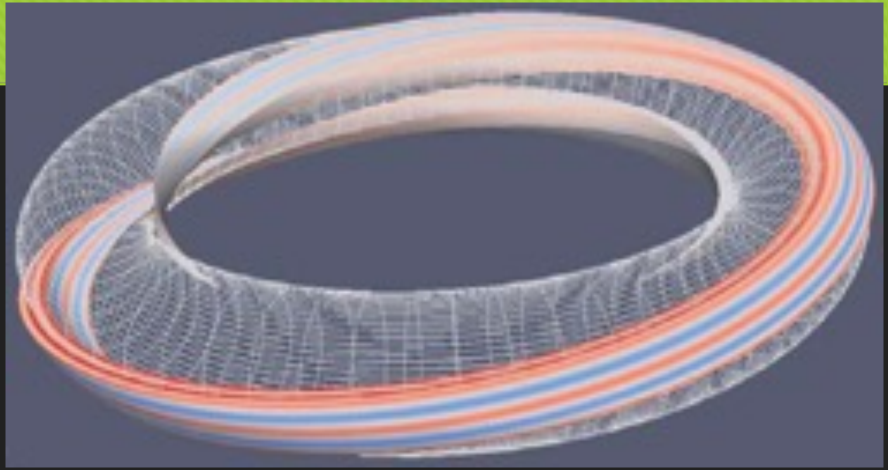
$$K_x = -\frac{\partial \ln B}{\partial y} + \frac{g^{xz} g^{xy} - g^{xx} g^{yz}}{B^2/c_b^2} \frac{\partial \ln B}{\partial z}$$

$$K_y = \frac{\partial \ln B}{\partial x} + \frac{g^{xz} g^{yy} - g^{xy} g^{yz}}{B^2/c_b^2} \frac{\partial \ln B}{\partial z}$$

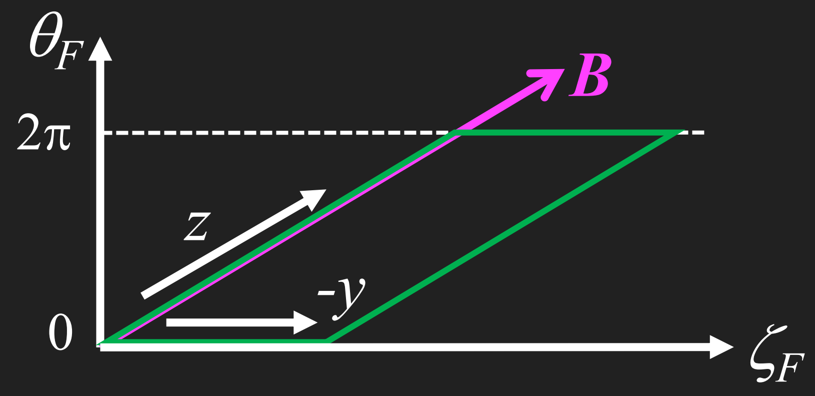
$$\frac{1}{L_{ns}} = -\frac{d \ln n_s}{dx}$$

$$\frac{1}{L_{Ts}} = -\frac{d \ln T_s}{dx}$$

Flux Tube Coordinate system



- Coordinate system
  - $x = c_x(\rho_F - \rho_0)$, $y = c_y[q(\rho_F)\,\theta_F - \zeta_F]$, $z = \theta_F$
  - $(\rho_F, \theta_F, \zeta_F)$: Arbitrary magnetic coordinates
  - $x \in [-L_x, +L_x]$, $y \in [-L_y, +L_y]$, $z \in [-N_\theta \pi, +N_\theta \pi]$ ($z = 0$ on the outward midplane)
- Fixed parameters for background distribution
  - Under local density/temperature gradients and magnetic shear $\Leftrightarrow$ $\rho^* \rightarrow 0$ limit
  - Periodic boundary conditions in radial direction



- Fourier rep. of perturbation quantities

$$A = A(x,y,z) = \sum_{k_\perp} \tilde{A}_{k_\perp}(z) e^{ik_x x + ik_y y}$$

$$= \sum_{k_\perp} \tilde{A}_{k_\perp}(\theta_F) e^{i(k_x + c_\theta \hat{s} \theta_F k_y) c_x(\rho_F - \rho_0) + ik_y c_y (q_0 \theta_F - \zeta_F)}$$

$$\hat{s} = \frac{\rho_0}{q_0}\frac{dq}{d\rho_F}, \quad c_\theta = \frac{c_y q_0}{\rho_0}$$

Boundary conditions in parallel direction

- Periodicity in the torus

$$A[x, y(\theta_F, \zeta_F), z(\theta_F)] = A[x, y(\theta_F + 2\pi, \zeta_F), z(\theta_F + 2\pi)]$$

- Boundary conditions in parallel direction

$$\tilde{A}_{\boldsymbol{k}_\perp}(z) = c_{k_y} \tilde{A}_{\boldsymbol{k}_\perp + \delta \boldsymbol{k}_\perp}(z + 2\pi)$$

$$\delta \boldsymbol{k}_\perp = -2\pi c_\theta \hat{s} k_y \nabla x \,, \; c_{k_y} = \exp\left(i 2\pi q_0 k_y c_y\right)$$

- Extension of z-space due to similarity to ballooning representation

$$\tilde{A}_{\boldsymbol{k}_\perp}(z) = c_{k_y} \tilde{A}_{\boldsymbol{k}_\perp + \delta \boldsymbol{k}_\perp}(z + 2N_\theta \pi)$$

$$\delta \boldsymbol{k}_\perp = -2N_\theta \pi c_\theta \hat{s} k_y \nabla x \,, \; c_{k_y} = \exp\left(i 2 N_\theta \pi q_0 k_y c_y\right)$$

- Constraints on aspect ratio

$$\left|\frac{\delta k_x}{k_{x,min}}\right| = \left|\frac{k_y}{k_{y,min}}\right| N_\theta m \,, \qquad m = 2\pi c_\theta \hat{s} \frac{k_{y,min}}{k_{x,min}}$$

Collision operators

- **Lenard-Bernstein model**

$$C_{sk}^{LB} = \nu_s \left[\frac{\partial}{\partial v_{||}} \left(v_{||} h_{sk} + v_{ts}^2 \frac{\partial h_{sk}}{\partial v_{||}} \right) + \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} \left(v_\perp^2 h_{sk} + v_{ts}^2 v_\perp \frac{\partial h_{sk}}{\partial v_\perp} \right) - k_\perp^2 \rho_{ts}^2 h_{sk} \right]$$

$$h_{sk} = f_{sk} + e_s F_{sM} J_{0sk} \phi_k / T_s$$

  - Momentum/energy is not conserved
  - (Numerical error suppressed if appropriate ν_s is set for velocity-space grid size)
- **Multi-particle species collision model** (Sugama+, 2009)
  - Particle, momentum and energy conservation, self-adjoint
  - Implicit version also available

In "gkvp_namelist"

Time integration methods	time_advnc= "*****"
Implicit method	"imp_colli",
4th Runge-Kutta-Gill	"rkg4"
Auto	"auto_init"

Normalizations in GKV

- Reference variables
  - Equilibrium distribution, unit of oscillation length in parallel direction ; L_{ref} (⇒ Major radius R_0)
  - Unit of oscillation length perpendicular direction ; Gyro-radius ρ_{ref}
  - Unit of velocity ; Thermal velocity v_{ref} , Unit of time ; L_{ref} / v_{ref}
  - Charge e, mass m_{ref} (⇒ proton m_{p}), number dens. n_{ref} (⇒ electron $n_{\mathrm{e}}(\rho_0)$), Temp. T_{ref} (⇒ main ion $T_{\mathrm{i}}(\rho_0)$)
    - For each species s , 、 m_s / m_{ref} , T_s / T_{ref} are given as reference variables.
    - $v_{\mathrm{ref}} = \sqrt{T_{\mathrm{ref}}/m_{\mathrm{ref}}}$
    - field strength is the value at the magnetic axis
- Normalization of Variables

$$\check{f}_{sk} = \frac{L_{\mathrm{ref}} v_{ts}^3}{\rho_{\mathrm{ref}} n_s} f_{sk} \,,\quad \check{\phi}_k = \frac{L_{\mathrm{ref}} e_{\mathrm{ref}}}{\rho_{\mathrm{ref}} T_{\mathrm{ref}}} \phi_k \,,\quad \check{A}_{\parallel k} = \frac{L_{\mathrm{ref}} e_{\mathrm{ref}}\, v_{\mathrm{ref}}}{\rho_{\mathrm{ref}}\, T_{\mathrm{ref}}} A_{\parallel k}$$

- Normalization of transport coefficient

$$\check{\chi} = \frac{L_{\mathrm{ref}}}{\rho_{\mathrm{ref}}^2 v_{\mathrm{ref}}} \chi = \chi / \chi^{GB}$$

In local model, $\rho_*^{-1} = L_{\mathrm{ref}} / \rho_{\mathrm{ref}}$ is indefinite

Dimensionless equations

$$\frac{\partial \check{f}_{sk}}{\partial \check{t}} + \check{v}_{ts}\check{v}_{\parallel}\check{\nabla}_{\parallel}\check{f}_{sk} + i\check{\boldsymbol{k}}\cdot\check{\boldsymbol{v}}_{sD}\check{f}_{sk} + \check{N}_{sk} - \check{v}_{ts}\check{\mu}\check{\nabla}_{\parallel}\check{B}\frac{\partial \check{f}_{sk}}{\partial \check{v}_{\parallel}}$$

$$= -\frac{\check{e}_s\check{F}_{sM}}{\check{T}_s}\left[\check{v}_{ts}\check{v}_{\parallel}\left(\check{\nabla}_{\parallel}J_{0sk}\check{\phi}_k + \frac{\partial J_{0sk}\check{A}_{\parallel k}}{\partial \check{t}}\right) + i\check{\boldsymbol{k}}\cdot\check{\boldsymbol{v}}_{sD}J_{0sk}\check{\phi}_k - i\check{\boldsymbol{k}}\cdot\check{\boldsymbol{v}}_{s*}J_{0sk}\left(\check{\phi}_k - \check{v}_{ts}\check{v}_{\parallel}\check{A}_{\parallel k}\right)\right]$$

$$\left[\check{\lambda}_D^2\check{k}_\perp^2 + \sum_s \frac{\check{e}_s^2\check{n}_s}{\check{T}_s}(1-\Gamma_{0sk})\right]\check{\phi}_k = \sum_s \check{e}_s\check{n}_s\int J_{0sk}\check{f}_{sk}\,d\check{v}^3$$

$$\check{k}_\perp^2\check{A}_{||k} = \check{\beta}\sum_s \check{e}_s\check{n}_s\int \check{v}_{ts}\check{v}_{\parallel}J_{0sk}\check{f}_{sk}\,d\check{v}^3$$

Debye length and beta-value

$$\check{\lambda}_D^2 = \frac{\lambda_{D,\mathrm{ref}}^2}{\rho_{\mathrm{ref}}^2}, \qquad \beta = \frac{v_{\mathrm{ref}}^2}{V_{A,\mathrm{ref}}^2} = \frac{\rho_{\mathrm{ref}}^2}{c^2/\omega_{p,\mathrm{ref}}^2}$$

Numerical scheme employed in GKV

- Time Integration
  - Fourth-order precision Runge-Kutta-Gill (with adjustable time increment width)
  - 2nd-order operator splitting + Crank-Nicholson for collision term implicit solving
- Spatial differentiation
  - (x, y) : Spectral method using FFT
  - z : 4th order central or 5th order up-wind
  - $(v_{\parallel}, \mu)$: 4th order central, $v_{\perp}$: Employ uniform grids
- Integration in velocity space
  - Trapezoidal formula + correction near $v_{\perp}$=0
- Required numerical library
  - FFTW is required, oothers provide interfaces for various libraries

Parallelization in GKV

- MPI domain decomposition
  - Decompose 6D field distribution in 6D space $(kx, ky, z, v_{\parallel}, \mu, s)$ to 5D space $(ky, z, v_{\parallel}, \mu, s)$
  - Decompose potential fluctuations in 3D space (kx, ky, z) to 2D space (ky, z)
  - 1 to 1 communication due to finite differences in $(z, v_{\parallel}, \mu)$
  - Reduction communication in $(v_{\parallel}, \mu, s)$ due to charge/current density calculations
  - Implicit collision scheme, Transpose communication due to FFT
  - Binary data is output as it is divided
  - Thread parallelization using OpenMP
  - Overlapping using master / slave thread communication

Brighten your ideas!

- We hope you will utilize the GKV Code in your research!
- We will continue to maintain/upgrade the GKV code in the future.
- GKV web page (you can download past materials) http://www.p.phys.nagoya-u.ac.jp/gkv/
- Latest source code https://github.com/GKV-developers/gkvp
- Usage Notes
  - The copyright of the code belongs to the developer team
  - Feel free to use it for non-commercial research
  - Correctness of the results is not guaranteed.
  - If you modify the distributed code and publish the results, please clearly state the modifications in your paper.
  - Please cite the following references in your paper using GKV

    Watanabe, T-H., and H. Sugama. "*Velocity–space structures of distribution function in toroidal ion temperature gradient turbulence*." Nuclear Fusion46.1 (2006): 24.